

GDC REVISION for MATH HL
for the CASIO model
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From [MENU] we use

- A. RUN-MAT
- B. EQUA
- C. STAT
- D. GRAPH
- E. RECUR (for sequences)

- **Remember**

- to use **RAD** or **DEG** appropriately in SETUP (shift-menu)
- in STAT – CALC to SET the first two lines
LIST1-LIST2 if you have frequencies
- in STAT – DIST we select
Data : **Variable** instead of List
For **NORMAL** we use ~~Npd~~ – **Ncd** – **InvN** (never Npd)
For **BINOM** we use ~~InvB~~ – **Bpd** – **Bcd** (never InvB)
For **POISSON** we use **Ppd** – **Pcd** – InvP
- in SETUP (shift-menu) select Derivative On

- **Common mistakes**

- For $x - 3$ we must use the difference – and not the symbol (-)
- For $\sin^2 x$ we must write $(\sin x)^2$
- For $\sin 3(x - 2)$ we must write $\sin(3(x - 2))$

In calculus, when we deal with sin, cos etc we must use **rad**

A. EXAMPLES FOR RUN-MAT

1. Use [MATH] to find the following

(a) $\log_2 5$

(b) $f'(2)$ and $f''(2)$ (by using $\frac{d}{dx}$ and $\frac{d^2}{dx^2}$) for

$$f(x) = \frac{2x + 5}{3x - 7}$$

(c) the definite integral

$$\int_3^5 \frac{2x + 5}{3x - 7} dx$$

(d) the sum

$$\sum_{k=7}^{20} \frac{2k + 5}{3k - 7}$$

(e) the sum of the multiples of 7 between 1 and 200

Answers

(a) 2.32 (b) -29 and -174 (c) 5.80 (d) 14.07

(e) Since $100/7=14.3$ and $200/7=28.6$ and the multiples of 7 have the form $7x$:

$$\sum_{x=15}^{28} 7x = 2107$$

2. Use [OPTION] and [PROB] to find (a) $12!$ (b) $\binom{12}{5}$ (c) $12P5$

Answers

(a) 479001600 (b) 792 (c) 95040

3. Use [OPTION] and [NUM]-[Abs] to find

(a) the definite integral

$$\int_3^5 |t^2 - 16| dt$$

(b) the sum

$$\sum_{k=1}^{30} |100 - 6k|$$

Answers

(a) 8 (b) 1358

4. Use [OPTION] – [CALC] – [SolveN] to solve the equations

(a) $e^x = 2x + 3$

(b) $\sin 3x = 0.5 \quad 0 \leq x \leq \pi$

(c) $\sin 3x = 0.5 \quad 0^\circ \leq x \leq 180^\circ$

Answers

(a) SolveN($e^x = 2x + 3$) gives: -1,37, 1,92

Notice: We can add a restriction for the domain. Use rad or degrees appropriately:

(b) SolveN($\sin 3x = 0.5, x, 0, \pi$) gives: $\pi/18, 5\pi/18, 13\pi/18, 17\pi/18$

(c) SolveN($\sin 3x = 0.5, x, 0, 180$) gives: 10, 50, 130, 170

5. Use [OPTION] – [CALC] – [SolveN] for probability density functions

Given that $f(x) = \frac{1}{1+x^4}, 0 \leq x \leq a$ is a pdf

(a) Find a

(b) Find Q_1

Answers

We must solve the equation $\int_0^a \frac{1}{1+x^4} dx = 1$ by using SolveN

It is a good idea to use a restriction for x as it takes time to find the solution.

(a) SolveN($\int_0^x \frac{1}{1+x^4} dx = 1, x, 0, "whatever"$) gives: $a = 1.396$

(b) SolveN($\int_0^x \frac{1}{1+x^4} dx = 0.25, x, 0, "whatever"$) gives: $Q_1 = 0.2502$

B. EXAMPLES FOR EQUA

6. Use [F2:Polynomial] to solve $x^3 - 5x^2 + 3x + 6 = 0$

Answer

3.79, 2, -0.791

7. Use [F1:Simultaneous] to solve the simultaneous equations

$$3x + 5y = 18$$

$$7x - 4y = -5$$

Answer

$x = 1, y = 3$

8. Use [F1:Simultaneous] to find

(a) the intersection of the three planes

$$3x + 6y - z = 19$$

$$x - 2y + 4z = 3$$

$$7x - 13z = -19$$

Answer Point (1,3,2)

(b) the intersection of the three planes

$$3x + 6y - z = 19$$

$$x - 2y + 4z = 3$$

$$5x + 2y + 7z = 25$$

Answer $(\frac{14}{3} - \frac{11}{6}z, \frac{5}{6} + \frac{13}{12}z, z)$. This is the line $r = \begin{pmatrix} 14/3 \\ 5/6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -11/6 \\ 13/12 \\ 1 \end{pmatrix}$

(c) the intersection of the two planes

$$3x + 6y - z = 19$$

$$x - 2y + 4z = 3$$

Answer Use (0 0 0) as a third equation. Thus

$(\frac{14}{3} - \frac{11}{6}z, \frac{5}{6} + \frac{13}{12}z, z)$. This is the line $r = \begin{pmatrix} 14/3 \\ 5/6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -11/6 \\ 13/12 \\ 1 \end{pmatrix}$

C. EXAMPLES FOR STAT

9. STATISTICS: Use [STAT] – [CALC] – [SET] – [VAR1]

A. Consider the data

2, 5, 7, 5, 3, 2, 5, 1, 7, 9

to find the following:

Mean = 4.6	Range = max-min = 9-1=8
Median = 5	Interquartile range = Q3 – Q1 = 7-2=5
Mode = 5	Standard deviation = σ = 2.458
Lower quartile = Q1 = 2	Variance = σ^2 = (2.458)² = 6.04
Upper quartile = Q3 = 7	

B. Consider the frequency table

x	10	20	30	40
frequency	3	5	7	5

to find the following:

Mean = 27	Range = max-min = 40-10=30
Median = 30	Interquartile range = Q3 – Q1 = 35-20=15
Mode = 30	Standard deviation = σ = 10.05
Lower quartile = Q1 = 20	Variance = σ^2 = (10.05)² = 101
Upper quartile = Q3 = 35	

C. Consider the frequency table with intervals

x	5-15	15-25	25-35	35-45
frequency	3	5	7	5

Use the midpoints (i.e. exactly the same data as above) to find:

Mean = 27	Standard deviation = σ = 10.05
Modal group = 25-35	Variance = σ^2 = (10.05)² = 101

Notice:

For Q1, Median, Q3 we do not use the GDC. We need the cumulative frequency graph

10. BINOMIAL DISTRIBUTION: Use [STAT] – [DIST] – [BINM]

The probability to win a game is 0.3

We play the game 12 times. So it is $B(n,p)$ with $n=12$ and $p=0.3$

Find the following probabilities

To win	Math expression	GDC	Result
exactly 5 times	$P(X = 5)$	Bpd(5)	0.158
at most 5 times	$P(X \leq 5)$	Bcd(0–5)	0.882
at least 5 times	$P(X \geq 5)$	Bcd(5–12)	0.276
less than 5 times	$P(X < 5)$	Bcd(0–4)	0.724
more than 5 times	$P(X > 5)$	Bcd(6–12)	0.118
	$P(3 < X \leq 8)$	Bcd(4–8)	0.506

Remark; Bcd(5-12) denotes Lower: 5, Upper: 12

11. POISSON DISTRIBUTION: Use [STAT] – [DIST] – [POISN]

The number of mistakes per page in a book follows Poisson distribution. There is on average 1 mistake per 5 pages

The frequency is 0.2 mistakes per page.

Find the probability that

- (a) There are 2 mistakes in 1 page
- (b) There are 4 mistakes in 10 pages
- (c) Among ten pages there are exactly 7 pages without mistakes

Answer

- (a) Poisson with $m=0.2$ $P(X=2)=0.0164$
- (b) Poisson with $m=2$, $P(X=4)=0.09022$
- (c) Poisson with $m=0.2$ for one page $P(X=0)=0.819$
And then Binomial with $n=10$, $p=0.819$, $P(Y=7)=0.176$

Otherwise, **Ppd** and **Pcd** work similarly as in the Binomial Distribution with **Bpd** and **Bcd** above.

12. NORMAL DISTRIBUTION: Use [STAT] – [DIST] – [NORM]

PROBLEM 1: FIND PROBABILITY so use [Ncd]

If mean is $\mu=1000$ and standard deviation is $\sigma=50$. Find the probability

That X is	Math notation	GDC	Result
Between 900 and 1035	$P(900 < X < 1035)$	Ncd(900–1035)	0.735
More than 1035	$P(X > 1035)$	Ncd(1035–999999)	0.242
Less than 900	$P(X < 900)$	Ncd(-99999–900)	0.0228
Exactly 975	$P(X = 975)$	0 (“exactly” is always 0)	

PROBLEM 2: PROBABILITY IS GIVEN so use [InvN]

If mean is $\mu=1000$ and standard deviation is $\sigma=50$

It is given	Find	Math notation	GDC	Result
Prob less than a is 0.35	a	$P(X < a) = 0.35$	InvN Tail: Left, Area: 0.35	$a = 981$
42% is more than b	b	$P(X > b) = 0.42$	InvN Tail: Right, Area: 0.42	$b = 1010$
	Q1 Q3	$P(Q1 < X < Q3) = 0.5$	InvN Tail: Central, Area: 0.5	Q1=966 Q3=1034

PROBLEM 3: μ or σ or both are unknown so use the formula $Z=(X-\mu)/\sigma$ and [InvN]

Suppose that $\mu = 800$ and σ is unknown

The information given can be expressed in different ways:

- The probability that X is less than **785** is **0.37**
- **37%** is less than **785**
- $P(X < 785) = 0.37$

Answer: We use the formula

$$Z = \frac{X - \mu}{\sigma}$$

Right hand side: we know $X = 785, \mu = 800$

Left hand side: Z is obtained by the GDC:

InvN
Tail: Left
Area=0.37
 $\sigma=1$
 $\mu=0$

$Z = -0.332$ and hence $\sigma = 45.18$

D. EXAMPLES FOR GRAPH

13. Let $f(x) = -x^2 + 3x + 6$

(a) Solve $f(x) = 0$

(i) by using [EQUA]

(ii) by using [SolveN]

(iii) by using [Graph]

(b) Solve $f(x) > 0$

by using [Graph] (only)

(c) Find the range of f

by using [Graph]

(d) Find the area of the region R enclosed by the curve $y = f(x)$ and the x-axis

(i) by using [MATH]

(ii) by using [GRAPH]

(e) Find the volume generated when the region R is rotated 2π rad in x-axis

by using [MATH] (only)

Answer

(a) $-1.372, 4.372$ (b) $-1.372 < x < 4.372$ (c) $y \leq 8.25$ (d) 31.595 (e) 655.1

14. Let $g(x) = 2x^3 - 15x^2 + 7x + 25$

(a) Solve $g(x) = 0$

(i) by using [EQUA]

(ii) by using [SolveN]

(iii) by using [Graph]

(b) Solve $g(x) > 0$

by using [Graph]

(c) Find the range of g

by using [Graph]

(d) Find the range of g if the domain is restricted to $-2 \leq x \leq 8$

by using [Graph]

(e) Find the area of the region R enclosed by the curve $y = f(x)$ and the x-axis

(i) by using [MATH]

(ii) by using [GRAPH]

Answer

(a) $-1.023, 1.824, 6.699$ (b) $-1.023 < x < 1.824, x > 6.699$ (c) $y \in R$
(d) $-65.84 \leq y \leq 145$ (e) $48.448 + 204.081 = 252.529$ (or directly the result)

15. Suppose that the **velocity** of moving body in terms of time is given by

$$v = 2t^3 - 15t^2 + 7t + 25$$

Use the graph of g above (exercise 14) to find

- (a) The velocity after 5 minutes
- (b) The time at which the velocity is minimum
- (c) The times when the direction changes
- (d) the displacement from the initial position in the first 5 seconds
- (e) the distance traveled in the first 5 seconds

Answer

- (a) -65 (b) $t = 4.75$ (c) $t = 1.824, t = 6.699$ (positive roots)
(d) -100 (e) $32.437 + 132.436 = 164.873$ (or directly the result)
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16. Let $f(x) = -x^2 + 3x + 6$ and $g(x) = 2x^3 - 15x^2 + 7x + 25$

- (a) Solve $f(x) = g(x)$
 - (i) by using [SolveN]
 - (ii) by using [Graph]
- (b) Solve $f(x) > g(x)$ only by using graphs:
 - (i) by using two graphs
 - (ii) by using one graph only [i.e. $f(x) - g(x)$]
- (c) Find the area enclosed by the two curves

Answer

- (a) $-0.9733, 1.51, 6.463$ (b) $x < -0.9733, 1.51 < x < 6.463$
(c) 232.51
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17. Let $f(x) = x^2 e^{0.002x} \sin 0.3x$

Find the tangent line and the normal line at $x = 2$

Answer

You can use [Graph] – [F4:Sketch] – [Tang] or [Norm]

Remember to [SETUP] – Derivative on

We find all the details below as well as the final answers

Point $(2, 2.268)$, $m_T = 3.2665$, $m_N = -0.306$

Tangent line: $y - 2.268 = 3.2665(x - 2)$ or $y = 3.2665x - 4.2654$

Normal line: $y - 2.268 = -0.306(x - 2)$ or $y = -0.306x + 2.8798$

E. EXAMPLES FOR RECUR

18. SEQUENCES: Use [RECUR]

Consider the sequence

$$10, 13, 16, 19, \dots$$

- (a) Find the 50th term
- (b) Find the sum of the first 50 terms
- (c) Find the first term that exceeds 200
- (d) Find the number of terms which are less than 200
- (e) Find the sum of the terms which are less than 200

Answer

The general term of this arithmetic sequence is $u_n = 10 + (n - 1) \times 3$

Use the following settings

[F3:TYPE] Select **F1** $a_n = 10 + (n - 1) \times 3$ (use the button F1 for n)

[F5:SET] Start: 1, End : 100

In order to get the sums as well, use SETUP (shift-menu)

Σ DISPLAY: On

Press **[EXE]** to get the lists for u_n and S_n . You can find all the answers

(a) 157 (b) 4175 (c) $u_{65} = 202$ (d) 64 (e) $S_{64} = 6688$

19. PERCENTAGE GROWTH: Use [RECUR]

The populations in cities A and B today are 100,000 and 150,000 respectively.

They increase by 3,1% and 2,3% per year respectively

Find

- (a) The population of the two cities after 12 years
- (b) After how many full years the population of city A exceeds 200,000
- (c) After how many full years the population of city A exceeds the one of B.

Answer

We use the formulas

$$a_n = 100000 \times (1.031)^n$$

$$b_n = 150000 \times (1.023)^n$$

Remember to remove the sums and start form 0 now

SETUP (shift-menu) **Σ DISPLAY: On**

[F5:SET] Start: 0, End : 100

(a) 144,246 and 197,060 (b) 23 years (c) 53 years