HAEF IB – FURTHER MATH HL

TEST

CALCULUS

by Christos Nikolaidis

Name:_____

Questions

1. *[Maximum mark: 5]*

Find

$$\lim_{x\to 0}\frac{e^x-1-x-0.5x^2}{x^3}$$ *[5 marks]*

2. *[Maximum mark: 15]*

(a) Solve the differential equation

$$e^{-3x}\frac{\mathrm{d}y}{\mathrm{d}x}+ye^{-3x}\tan x=\cos x \qquad \text{for } x\in\left[0,\frac{\pi}{2}\right[$$ *[8 marks]*

given that $y=1$ when $x=0$. Give your answer in the form $y=f(x)$.

(b) Use Euler's method with a step length of $h=0.2$ to find an approximation to the value of y when $x=1$. *[5 marks]*

(c) Show that the error of the approximation found in question (b) is less than 0.3. *[2 marks]*

3. *[Maximum mark: 9]*

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(9x+y)(4x+y)}{x^2}$ for $x>0$, given that $y=1$ when $x=1$.

(a) Show that this can be written in the form $\frac{\mathrm{d}y}{\mathrm{d}x}=f\left(\frac{y}{x}\right)$. *[1 mark]*

(b) Hence solve the equation, giving the answer in the form $y=f(x)$. *[8 marks]*

4. *[Maximum mark: 7]*

Determine the convergence or divergence of the following series:

(a) $\sum_{n=1}^{\infty} \frac{2n+1}{3n+4}$. *[2 marks]*

(b) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$. *[5 marks]*

5. *[Maximum mark: 15]*

(a) Show that $\int_{1}^{\infty} xe^{-x}dx = \frac{2}{e}$. *[7 marks]*

(b) Determine the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{n}{e^n}$. *[3 marks]*

(ii) $\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)e^n}$, by using the limit comparison test. *[5 marks]*

6. *[Maximum mark: 9]*

Consider the infinite series $\sum_{n=1}^{\infty} \frac{(0.4)^n}{n} x^n$

(a) Find the **radius** of convergence. *[4 marks]*

(b) Find the **interval** of convergence. *[3 marks]*

(c) **Hence** find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(0.4)^n}{n}(x-3)^n$. *[2 marks]*