HAEF IB – FURTHER MATH HL

TEST 3

SETS, GROUPS AND RELATIONS

PAPER 1

by Christos Nikolaidis

P1:___/35	P2:___/35

Total: _____%

Grade:

Name:____________________________________

Date: 25/1/2017

Questions

1. *[Maximum mark: 5]*

(a) Show by means of a Venn diagram that $X - Y = X \cap Y'$ *[1 mark]*

(b) Using (a) and set algebra, prove that $A - (B \cup C) = (A - B) \cap (A - C)$ *[4 marks]*

2. *[Maximum mark: 7]*

Consider the function $f : R^{+} \times R \to R \times R^{+}$ given by

$$f(x, y) = (\ln x, e^{x+y})$$

(a) Show that f is a bijection *[6 marks]*

(b) Find f^{-1} *[1 mark]*

3. *[Maximum mark: 8]*

Consider the functions $f : A \to B$ and $g : B \to C$. Given that $g \circ f : A \to C$ is a bijection, show that

(a) f is an injection *[3 marks]*

(b) g is a surjection *[3 marks]*

(c) f and g are not necessarily bijections. *[2 marks]*

4. *[Maximum mark: 15]*

Let $D = R - \{1\}$ and $f : D \to D$ a function given by

$$f(x) = \frac{x+1}{x-1}$$

(a) Explain why f is a bijection. *[2 marks]*

(b) Show that f is self-inverse *[2 marks]*

(c) Let T be a relation on D given by

$$x T y \quad \text{if and only if} \quad y = f(x)$$

Determine whether T is reflexive, symmetric or transitive. *[5 marks]*

(d) Let S be a relation on $D \times R$ such that

$$(x, y) \;\; S \;\; (a, b) \quad \text{if and only if} \quad y + f(a) - b = f(x)$$

(i) Show that S is an equivalence relation.

(ii) Describe the equivalence classes of S (i.e. the partition of $D \times R$) *[6 marks]*

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PAPER 2

by Christos Nikolaidis

Name:____________________________________

Date: 25/1/2017

Questions

1. *[Maximum mark: 15]*

Consider the binary operation

$$x * y = 5xy$$

on the set of non-zero real numbers R^*.

(a) Show that $(R^*, *)$ has an identity element a and state its value. *[2 marks]*

(b) Show that $(R^*, *)$ is an Abelian group. *[5 marks]*

Consider also a homomorphism

$$f:(R^*,*)\to(R,+)$$

where $(R,+)$ is the standard additive group.

(c) Show that $f(a)=0$. *[2 marks]*

(d) Given that $f(x)=\ln|k\,x|$, where k is a positive integer

(i) find the value k, by using (c)

(ii) confirm that f is a homomorphism;

(iii) explain why f is not an isomorphism;

(iv) find the kernel $Ker\,f$.

(v) Describe the cosets of $Ker\,f$

[6 marks]

2. *[Maximum mark: 20]*

Consider the multiplicative group $(Z_7^*, \times_7)$, where $Z_7^* = \{1,2,3,4,5,6\}$ and $\times_7$ is the multiplication of integers modulo 7.

(a) Write down the Cayley table of this group. *[4 marks]*

(b) Show that $(Z_7^*, \times_7)$ is cyclic and find its smallest generator. *[3 marks]*

Consider also the additive group $(Z_6, +_6)$, where $Z_6 = \{0,1,2,3,4,5\}$ and $+_6$ is the addition of integers modulo 6.

(c) If f is a homomorphism from $(Z_7^*, \times_7)$ to $(Z_6, +_6)$, with $f(3) = 1$

(i) Find the value of $f(2)$ by using the fact $3 \times_7 3 = 2$

(ii) Copy and complete the following tables by applying f on the powers of 3

x	1	2	3	4	5	6
$f(x)$	0		1			

[6 marks]

(d) If g is a homomorphism from $(Z_7^*, \times_7)$ to $(Z_6, +_6)$, with $g(3) = 2$, copy and complete the following table

x	1	2	3	4	5	6
$g(x)$			2			

[4 marks]

[2 marks]

(e) Determine which of the two functions f, g is an isomorphism. Explain. *[1 marks]*

(f) Write down the kernel of g.