

INTERNATIONAL BACCALAUREATE  
MATH HL

**OPTION CALCULUS**  
**EXERCISES**  
**2. DIFFERENTIAL EQUATIONS OF SEPARABLE VARIABLES**

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**Past Paper Questions**

1. Given that  $\frac{dy}{dx} = e^x - 2x$  and  $y = 3$  when  $x = 0$ , find an expression for  $y$  in terms of  $x$ .  

**(Total 6 marks)**
2. Solve the differential equation  $\frac{dy}{dx} = 2xy^2$  given that  $y = 1$  when  $x = 0$ . Give your answer in the form  $y = f(x)$ .  

**(Total 6 marks)**
3. Solve the differential equation  $x\frac{dy}{dx} - y^2 = 1$ , given that  $y = 0$  when  $x = 2$ . Give your answer in the form  $y = f(x)$ .  

**(Total 6 marks)**
4. Solve the differential equation  $xy\frac{dy}{dx} = 1 + y^2$ , given that  $y = 0$  when  $x = 2$ .  

**(Total 3 marks)**
5. Find the general solution of the differential equation  $\frac{dx}{dr} = kx(5 - x)$  where  $0 < x < 5$ , and  $k$  is a constant.  

**(Total 3 marks)**
6. Solve the differential equation  $(x^2 + 1)\frac{dy}{dx} - xy = 0$  where  $x > 0$ ,  $y > 0$ , given that  $y = 1$  when  $x = 1$ .  

**(Total 6 marks)**
7. Solve the differential equation  $(x + 2)^2\frac{dy}{dx} = 4xy$  ( $x > -2$ ), given that  $y = 1$  when  $x = -1$ .  

**(Total 6 marks)**

8. Solve the differential equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ , given that  $y = \sqrt{3}$  when  $x = \frac{\sqrt{3}}{3}$ .

Give your answer in the form  $y = \frac{ax + \sqrt{a}}{a - x\sqrt{a}}$  where  $a \in \mathbb{Z}^+$ .

**(Total 6 marks)**

9. The equation of motion of a particle with mass  $m$ , subjected to a force  $kx$  can be written as  $kx = mv \frac{dv}{dx}$ , where  $x$  is the displacement and  $v$  is the velocity. When  $x = 0$ ,  $v = v_0$ . dx Find  $v$ , in

**(Total 3 marks)**

10. A sample of radioactive material decays at a rate which is proportional to the amount of material present in the sample. Find the half-life of the material if 50 grams decay to 48 grams in 10 years.

**(Total 3 marks)**

11. The tangent to the curve  $y = f(x)$  at the point  $P(x, y)$  meets the  $x$ -axis at  $Q(x - 1, 0)$ . The curve meets the  $y$ -axis at  $R(0, 2)$ . Find the equation of the curve.

**(Total 6 marks)**

12. When air is released from an inflated balloon it is found that the rate of decrease of the volume of the balloon is proportional to the volume of the balloon. This can be represented by the differential equation  $\frac{dv}{dt} = -kv$ , where  $v$  is the volume,  $t$  is the time and  $k$  is the constant of proportionality.

(a) If the initial volume of the balloon is  $v_0$ , find an expression, in terms of  $k$ , for the volume of the balloon at time  $t$ .

(b) Find an expression, in terms of  $k$ , for the time when the volume is  $\frac{v_0}{2}$ .

**(Total 4 marks)**